

TWO-PHOTON DECAY RATES OF TRUE NEUTRAL PSEUDOSCALAR MESONS FROM DATA ON THEIR TRANSITION FORM FACTORS

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By means of the universal unitary and analytic model of electromagnetic structure of hadrons the two-photon decay rates of $P = \pi^0, \eta, \eta'$ mesons are determined in an alternative way from data on their transition form factors.

I. INTRODUCTION

There are three true neutral particles ($P = \pi^0, \eta, \eta'$) from the nonet of pseudoscalar mesons, which decay into 2γ . The $\pi^0 \rightarrow 2\gamma$ channel with branching ratio $(98.798 \pm 0.032)\%$, $\eta \rightarrow 2\gamma$ channel with branching ratio $(39.31 \pm 0.20)\%$ and $\eta' \rightarrow 2\gamma$ channel with branching ratio $(2.10 \pm 0.12)\%$, corresponding to the following decay widths: $\Gamma_{exp}(\pi^0 \rightarrow 2\gamma) = (7.84 \pm 0.56)eV$, $\Gamma_{exp}(\eta \rightarrow 2\gamma) = (511.03 \pm 27.79)eV$ and $\Gamma_{exp}(\eta' \rightarrow 2\gamma) = (4305.00 \pm 424.95)eV$, respectively.

They are an average of repeated measurements at different experiments exploiting the Primakoff effect and $e^+e^- \rightarrow e^+e^-P$ process; in the case of $\eta'(958)$ also $e^+e^- \rightarrow e^+e^-\pi^+\pi^-\gamma$, $e^+e^- \rightarrow e^+e^-\eta\pi^+\pi^-$ and $e^+e^- \rightarrow e^+e^-\eta\pi^0\pi^0$ processes.

In this contribution we demonstrate a more effective method of a determination of 2γ decay widths of π^0 , η and η' . More concretely, by a description of existing data on the corresponding transition form factors (FFs) in space-like and time-like regions simultaneously with the elaborate in [1] Unitary and Analytic ($U\&A$) model of π^0 , η and η' transition FFs.

The main idea consists in the following: The behavior of the meson-photon transition FF $F_{\gamma P}(Q^2)$ for $Q^2 \rightarrow 0$

$$\lim_{Q^2 \rightarrow 0} F_{\gamma P}(Q^2) = \frac{1}{4\pi^2 f_P} \quad (1)$$

can be determined from the axial anomaly in the chiral limit of QCD, where f_P is the meson weak decay constant and $Q^2 = -q^2 = -t$. However, in order to take into account the fact that f_η and $f_{\eta'}$ (unlike $f_\pi = 93MeV$) are not directly measurable quantities, employing the relation for the two-photon partial width

$$\Gamma(P \rightarrow \gamma\gamma) = \frac{\alpha^2}{64\pi^3 f_P^2} m_P^2 \quad (2)$$

of the pseudoscalar meson P , one comes to a redefinition of the FF norm

$$F_{\gamma P}(0) = \frac{2}{\alpha m_P} \sqrt{\frac{\Gamma(P \rightarrow \gamma\gamma)}{\pi m_P}} \quad (3)$$

to be expressed through the $\Gamma(P \rightarrow \gamma\gamma)$. So, fitting all existing data on $F_{\gamma\pi^0}(t)$, $F_{\gamma\eta}(t)$, $F_{\gamma\eta'}(t)$ in space-like and time-like regions by the sophisticated $U\&A$ models simultaneously, one finds normalization points values $F_{\gamma\pi^0}(0)$, $F_{\gamma\eta}(0)$, $F_{\gamma\eta'}(0)$ and then, finally, by means of (3) the most reliable values of $\Gamma(\pi^0 \rightarrow \gamma\gamma)$, $\Gamma(\eta \rightarrow \gamma\gamma)$ and $\Gamma(\eta' \rightarrow \gamma\gamma)$.

II. EXPERIMENTAL DATA ON PSEUDOSCALAR MESON TRANSITION FORM FACTORS

One of the first measurements of π^0 , η and η' transition FFs in the space-like region was carried out by CELLO Colab. [2], where really the π^0 transition FF in the space-like region was observed for the first time. An extension

of the above-mentioned measured interval to higher values of Q^2 was achieved by CLEO Collab. [3] to be recently supplemented for π^0 up to $Q^2 = 34.36 \text{ GeV}^2$ by BABAR Collab. [4]. For measurements of π^0 , η and η' transition FFs in the time-like region commonly the annihilation processes $e^+e^- \rightarrow \gamma P$ are used. Especially for π^0 and η a lot of data was obtained on colliding $e^+ - e^-$ beams in Novosibirsk by SND detector [5], [6] and by CMD-2 detector for η transition FF [7] to be corrected later on and published together with π^0 transition FF data in [8].

Nevertheless, also here 1/3 of the presented data on $\sigma_{tot}(e^+e^- \rightarrow \gamma\gamma)$ gives zero information on $F_{\gamma\eta}(t)$ as there are only upper boundary estimations, or the values to be charged by the error equal, even larger, than the central value of the cross-section and this collection of data have had to be carefully analyzed before our application. So, finally we are left with reliable (see Figs 1-3):

- 81 exp. points on π^0 transition FF from the interval $-34.36 \text{ GeV}^2 \leq t \leq 1.3535 \text{ GeV}^2$;
- 58 exp. points on η transition FF from the interval $-12.74 \text{ GeV}^2 \leq t \leq 1.0685 \text{ GeV}^2$;
- 56 exp. points on η' transition FF from the interval $-15.3 \text{ GeV}^2 \leq t \leq 0.48 \text{ GeV}^2$,

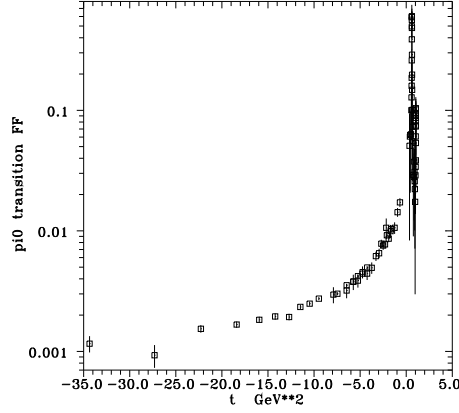


FIG. 1: The data on $\gamma - \pi^0$ transition form factor.

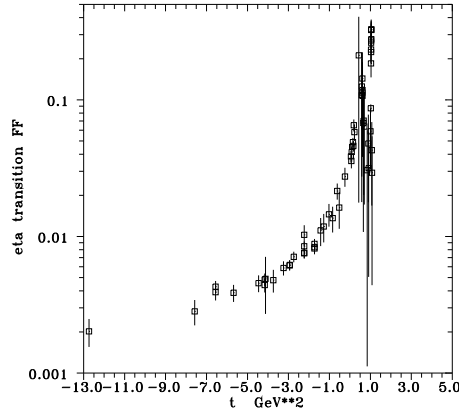


FIG. 2: The data on $\gamma - \eta$ transition form factor.

by means of which and the corresponding $U\&A$ models will be $\Gamma(P \rightarrow \gamma\gamma)$ determined.

III. UNITARY AND ANALYTIC MODEL OF PSEUDOSCALAR MESON TRANSITION FORM FACTORS

There is single FF for each $\gamma^* \rightarrow \gamma P$ transition to be defined by a parametrization of the matrix element of the EM current $J_\mu^{EM} = 2/3 \bar{u}\gamma_\mu u - 1/3 \bar{d}\gamma_\mu d - 1/3 \bar{s}\gamma_\mu s$

$$\langle P(p)\gamma(k) | J_\mu^{EM} | 0 \rangle = \epsilon_{\mu\nu\alpha\beta} p^\nu \epsilon^\alpha k^\beta F_{\gamma P}(q^2), \quad (4)$$

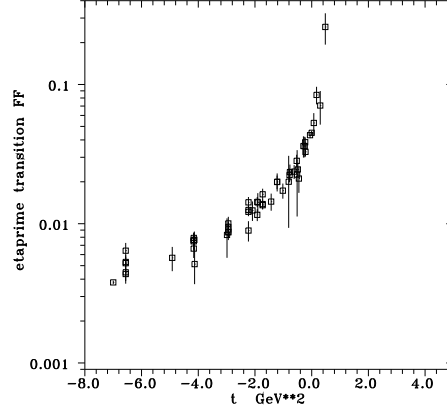


FIG. 3: The data on $\gamma - \eta'$ transition form factor.

where ϵ^α is the polarization vector of γ , $\epsilon_{\mu\nu\alpha\beta}$ appears as only the pseudoscalar meson belongs to the abnormal spin-parity series.

A straightforward calculation of $F_{\gamma P}(Q^2)$ behavior in QCD is impossible and one can obtain [9] in the framework of PQCD only the asymptotic behavior

$$\lim_{Q^2 \rightarrow \infty} Q^2 F_{\gamma P}(Q^2) = 2f_P \quad (5)$$

where f_P is already explained the meson weak decay constant.

Our intention is to achieve optimal description of all $t < 0$ and $t > 0$ data on $F_{\gamma\pi^0}(t)$, $F_{\gamma\eta}(t)$, $F_{\gamma\eta'}(t)$ always by one, however, distinct for π^0 , η , η' , analytic function explicitly known on the real axis of t -plane from $-\infty$ to $+\infty$, respecting all known properties of $F_{\gamma P}(t)$ like

- the asymptotic behavior (5)
- the normalization (1)
- the reality condition $F_{\gamma P}^*(t) = F_{\gamma P}(t^*)$
- analytic properties with the lowest branch point at $t_0 = m_{\pi^0}^2$
- unitarity condition, i.e. $Im F_{\gamma P}(t) \neq 0$ only from the lowest branch point on the positive real axis of t -plane to $+\infty$.

The transition FF $F_{\gamma P}(t)$ is suitable to split into two terms depending on the isotopic character of the photon

$$F_{\gamma P}(t) = F_{\gamma P}^{I=0}(t) + F_{\gamma P}^{I=1}(t) \quad (6)$$

where $F_{\gamma P}^{I=0}(t)$ can be saturated by only isoscalar vector mesons and $F_{\gamma P}^{I=1}(t)$ can be saturated by only isovector vector mesons, whereby both sets possess photon quantum numbers.

How much resonances will be considered? It is prescribed by the interval of data in $t > 0$ region. The data on π^0 transition FF allow naturally to consider all three ground state vector mesons $\rho(770)$, $\omega(782)$, $\phi(1020)$ and also $\omega'(1420)$ and $\rho'(1450)$, in order to obtain automatically normalized models for $F_{\gamma P}^{I=0}(t)$ and $F_{\gamma P}^{I=1}(t)$.

With the aim of obtaining comparable results for all three cases, the same number of resonances is conserved also for η and η' transition FFs and resonance parameters are fixed at the TABLE values.

Then one can write down the following automatically normalized five resonance VMD parametrization of transition FFs

$$\begin{aligned} F_{\gamma P}^{I=0}(t) &= \frac{1}{2} F_{\gamma P}(0) \frac{m_\omega'^2}{m_\omega'^2 - t} \\ &+ \left\{ \frac{m_\omega^2}{m_\omega^2 - t} - \frac{m_\omega'^2}{m_\omega'^2 - t} \right\} (f_{\gamma P\omega} / f_\omega) \\ &+ \left\{ \frac{m_\phi^2}{m_\phi^2 - t} - \frac{m_\omega'^2}{m_\omega'^2 - t} \right\} (f_{\gamma P\phi} / f_\phi) \end{aligned}$$

$$F_{\gamma P}^{I=1}(t) = \frac{1}{2}F_{\gamma P}(0)\frac{m_{\rho'}^2}{m_{\rho}^2 - t} + \left\{\frac{m_{\rho}^2}{m_{\rho}^2 - t} - \frac{m_{\rho'}^2}{m_{\rho}^2 - t}\right\}(f_{\gamma P\rho}/f_{\rho})$$

suitable for a construction of $U\&A$ models of $F_{\gamma\pi^0}(t)$, $F_{\gamma\eta}(t)$, $F_{\gamma\eta'}(t)$ transition FFs.

The analytic properties of $F_{\gamma P}(t)$ consist in the assumption, that $F_{\gamma P}(t)$ is analytic in the whole complex t -plane besides the cut on the positive real axis from $t_0 = m_{\pi^0}^2$ up to $+\infty$, as there is the intermediate $\pi^0\gamma$ state allowed in the unitarity condition of every π^0 , η and η' transition FF generating just the lowest branch point $t_0 = m_{\pi^0}^2$. On the other hand, just from the unitarity condition it follows that there is an infinite number of higher branch points on the positive real axis as there is allowed infinite number of higher intermediate states in the unitarity condition of FFs under consideration. In our model we restrict to two square-root cut approximation of the latter picture, practically to be realized by an application of the following nonlinear transformations

$$t = t_0 - \frac{4(t_{in}^s - t_0)}{[1/V - V]^2} \quad (7)$$

$$t = t_0 - \frac{4(t_{in}^v - t_0)}{[1/W - W]^2} \quad (8)$$

respectively, and subsequently also nonzero values of vector-meson widths, $\Gamma_s \neq 0$ and $\Gamma_v \neq 0$, are established.

The inelastic square-root branch points t_{in}^s and t_{in}^v include in average contributions of all higher important thresholds effectively and are left to be free parameters of the constructed model. Variable V (W) is conformal mapping

$$V(t) = i \frac{\sqrt{q_{in}^s + q} - \sqrt{q_{in}^s - q}}{\sqrt{q_{in}^s + q} + \sqrt{q_{in}^s - q}} \quad (9)$$

$$q = [(t - t_0)/t_0]; \quad q_{in}^s = [(t_{in}^s - t_0)/t_0]$$

of the four-sheeted Riemann surface in t -variable into one V -plane (W -plane). As a result of application of the nonlinear transformations (7) and (8) all VMD terms first give the factorized forms

$$\frac{m_i^2}{(m_i^2 - t)} = \left(\frac{1 - V^2}{1 - V_N^2}\right) \frac{(V_N - V_{i0})(V_N + V_{i0})(V_N - 1/V_{i0})(V_N + 1/V_{i0})}{(V - V_{i0})(V + V_{i0})(V - 1/V_{i0})(V + 1/V_{i0})}$$

on the pure asymptotic term $(\frac{1-V^2}{1-V_N^2})$, independent on the flavour of vector-mesons under consideration (however it depends on the isospin) and carrying just the asymptotic behavior of VMD model, and the so-called resonant term (the second one) describing the resonant structure of VMD terms, which, however, for $|t| \rightarrow \infty$ is going out on the real constant. The subindex 0 means that still $\Gamma = 0$ of all vector mesons is considered. In order to demonstrate

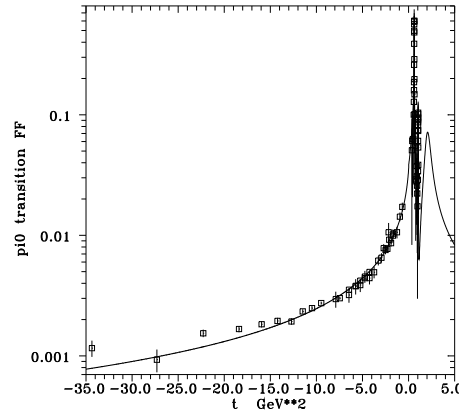


FIG. 4: A description of data on $\gamma - \pi^0$ transition form factor.

the reality condition $F_{\gamma P}^*(t) = F_{\gamma P}(t^*)$ explicitly, one can utilize relations between complex conjugate values of the corresponding zero-width VMD model pole positions in $V(W)$ plane

$$V_{\omega 0} = -V_{\omega 0}^*, \quad W_{\rho 0} = -W_{\rho 0}^* \quad (10)$$

$$V_{i0} = 1/V_{i0}^*, \quad i = \phi, \omega', \quad W_{\rho'0} = 1/W_{\rho'0}^* \quad (11)$$

following from the experience that in a fitting procedure of existing data on $F_{\gamma P}(t)$ such numerical value of $t_{in}^s, (t_{in}^v)$ is found that

$$(m_i^2 - \Gamma_i^2/4) < t_{in}^s, t_{in}^v \quad i = \omega, \rho \quad (12)$$

$$(m_j^2 - \Gamma_j^2/4) > t_{in}^s, t_{in}^v \quad j = \phi, \omega', \rho'. \quad (13)$$

Finally, incorporating $\Gamma \neq 0$ by a substitution

$$m_r^2 \rightarrow (m_r - i\Gamma_r/2)^2 \quad (14)$$

one comes to $U\&A$ model of $F_{\gamma P}(t)$ in the form

$$F_{\gamma P}^{I=0}[V(t)] = \left(\frac{1-V^2}{1-V_N^2}\right)^2 \left\{ \frac{1}{2} F_{\gamma P}(0) H(\omega') \right. \\ \left. + [L(\omega) - H(\omega')] a_\omega \right. \\ \left. + [H(\phi) - H(\omega')] a_\phi \right\}$$

$$F_{\gamma P}^{I=1}[W(t)] = \left(\frac{1-W^2}{1-W_N^2}\right)^2 \left\{ \frac{1}{2} F_{\gamma P}(0) H(\rho') \right. \\ \left. + [L(\rho) - H(\rho')] a_\rho \right\}$$

with

$$L(\omega) = \frac{(V_N - V_\omega)(V_N - V_\omega^*)(V_N - 1/V_\omega)(V_N - 1/V_\omega^*)}{(V - V_\omega)(V - V_\omega^*)(V - 1/V_\omega)(V - 1/V_\omega^*)}$$

$$H(i) = \frac{(V_N - V_i)(V_N - V_i^*)(V_N + V_i)(V_N + V_i^*)}{(V - V_i)(V - V_i^*)(V + V_i)(V + V_i^*)}, i = \phi, \omega'$$

$$L(\rho) = \frac{(W_N - W_\rho)(W_N - W_\rho^*)(W_N - 1/W_\rho)(W_N - 1/W_\rho^*)}{(W - W_\rho)(W - W_\rho^*)(W - 1/W_\rho)(W - 1/W_\rho^*)}$$

$$H(\rho') = \frac{(W_N - W_{\rho'})(W_N - W_{\rho'}^*)(W_N + W_{\rho'})(W_N + W_{\rho'}^*)}{(W - W_{\rho'})(W - W_{\rho'}^*)(W + W_{\rho'})(W + W_{\rho'}^*)}$$

and normalization points $V(t)_{t=0} = V_N, W(t)_{t=0} = W_N$. It depends on the following five free parameters

$$t_{in}^s, t_{in}^v, a_j = (f_{\gamma P j} / f_j) \quad j = \rho, \omega, \phi \quad (15)$$

which are found in an optimal description of existing data on π^0, η and η' transition FFs, presented in previous Figs. 1-3.

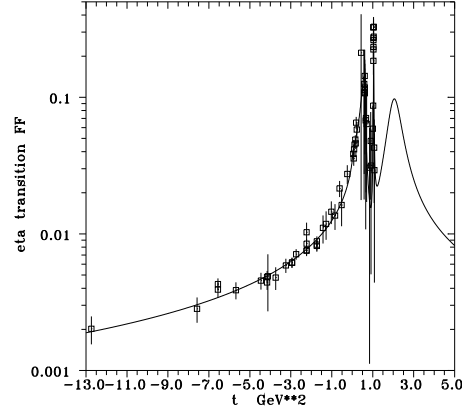


FIG. 5: A description of data on $\gamma - \eta$ transition form factor.

IV. RESULTS

The constructed $U\&A$ model has been applied to optimal description of existing data and the results are as follows: for π^0 : (see Fig.4)

$$\begin{aligned} q_{in}^s &= 5.5210 \pm 0.0084; & q_{in}^v &= 5.6120 \pm 0.1414; \\ a_\omega &= 0.0063 \pm 0.0013; & a_\phi &= -0.0004 \pm 0.0001; \\ a_\rho &= 0.0212 \pm 0.0006; & F_{\gamma\pi^0}(0) &= 0.0352 \pm 0.0070; [m_\pi^{-1}] \quad \chi^2/ndf = 121/75 = 1.61, \end{aligned}$$

for η : (see Fig.5)

$$\begin{aligned} q_{in}^s &= 6.7104 \pm 0.0190; & q_{in}^v &= 5.5006 \pm 0.0632; \\ a_\omega &= 0.0002 \pm 0.0014; & a_\phi &= -0.0020 \pm 0.0003; \\ a_\rho &= 0.0250 \pm 0.0013; & F_{\gamma\eta}(0) &= 0.0348 \pm 0.0026; [m_\pi^{-1}] \quad \chi^2/ndf = 52/52 = 1.00 \end{aligned}$$

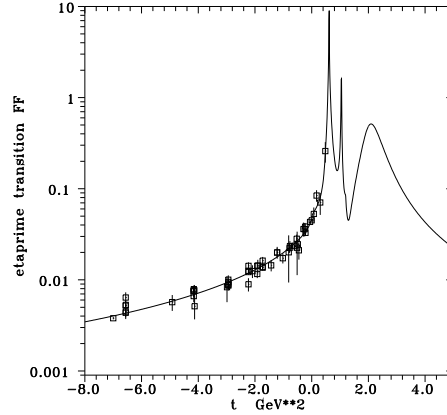


FIG. 6: A description of data on $\gamma - \eta'$ transition form factor.

for η' : (see Fig.6)

$$\begin{aligned} q_{in}^s &= 5.5366 \pm 0.0891; & q_{in}^v &= 7.7554 \pm 0.0158; \\ a_\omega &= -0.1134 \pm 0.0078; & a_\phi &= 0.0098 \pm 0.0091; \\ a_\rho &= 0.1241 \pm 0.0026; & F_{\gamma\eta'}(0) &= 0.0469 \pm 0.0016[m_\pi^{-1}]; \quad \chi^2/ndf = 59/50 = 1.18 \end{aligned}$$

Finally, recalculated values of two-photon decay widths from the obtained normalization points $F_{\gamma P}(0)$ are $\Gamma(\pi^0 \rightarrow \gamma\gamma) = (5.28 \pm 0.26)eV$, $\Gamma(\eta \rightarrow \gamma\gamma) = (428.33 \pm 63.70)eV$ and $\Gamma(\eta' \rightarrow \gamma\gamma) = (4142.88 \pm 274.01)eV$, to be compared with TABLE values $\Gamma_{exp}(\pi^0 \rightarrow \gamma\gamma) = (7.84 \pm 0.56)eV$, $\Gamma_{exp}(\eta \rightarrow \gamma\gamma) = (511.03 \pm 27.79)eV$ and $\Gamma_{exp}(\eta' \rightarrow \gamma\gamma) = (4305.00 \pm 424.95)eV$, respectively.

V. CONCLUSIONS

By an alternative method we have determined two-gamma decay widths of π^0 , η and η' pseudoscalar mesons. The results are differing from the TABLE values, though our results for π^0 and η' are more precise. We are not qualified to say, what results are more true. What can we say, that if more precise data on π^0 , η and η' transition FFs are measured, more precise values of pseudoscalar mesons decay two-photon widths are obtained by our method.

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